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TECHNICAL NOTE 3601

PRESSURE RISE ASSOCIATED WITH SHOCK-INDUCED BOUNDARY-LAYER SEPARATION

By Eugene S. Love

Langley Aeronautical Laboratory
Langley Field, Va.

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SUMMARY

Some recent contributions to the problem of shock-induced separation of the boundary layer are examined, and additional analytical and experimental results are presented. The probable ranges of pressure rises and flow deflections associated with separation are indicated. Consideration is given to the effects of Mach number, adverse pressure gradient, and Reynolds number for laminar boundary layers and to the effects of Mach number, Reynolds number, and ratio of specific heats for turbulent boundary layers.

INTRODUCTION

In recent years, the phenomena associated with shock-induced separation of the boundary layer have received increased attention because of the important influence that separation may have upon the overall aerodynamic characteristics of complete aircraft configurations by affecting the performance of individual components.

The purpose of the present study is to examine some of the recent contributions to the problem with a view toward facilitating practical application and to present additional analytical and experimental results. The first part of the paper is concerned with laminar boundary layers and the second part is concerned with turbulent boundary layers.

SYMBOLS

M   Mach number

U   velocity immediately outside boundary layer

q   dynamic pressure

p static pressure

δ^* displacement thickness for incompressible flow

θ momentum thickness for incompressible flow

$H \equiv \frac{\delta^*}{\theta}$

γ ratio of specific heats

$K \equiv \frac{M_1}{M_o}$

x longitudinal coordinate

R Reynolds number

P pressure-rise coefficient, $\frac{p - p_o}{q_o}$

$\epsilon \equiv \frac{p_1}{p_o}$

ϕ total two-dimensional turning angle through a gradual compression or oblique shock

F nondimensional velocity gradient where x_o is distance from leading edge to beginning of adverse velocity gradient, $\frac{x_o}{U_o} \frac{dU}{dx}$

Subscripts:

x value at distance x from leading edge

step value at step location

max maximum

o undisturbed free stream or initial value

1	condition behind compression or shock that emerges from boundary layer in shock-induced separation
i	incompressible
c	compressible
f	first peak for initially turbulent boundary layer or first peak downstream of laminar foot for initially laminar boundary layer
s	value causing separation
sp	value at separation point

LAMINAR BOUNDARY LAYERS

Analytical Considerations

In reference 1, Von Doenhoff calculated by use of the Von Karman-Millikan method for incompressible flow (ref. 2) the ratio of the velocity outside the boundary layer at the separation point to the undisturbed free-stream velocity U_s/U_o as a function of a nondimensional velocity gradient F_i for the condition of uniformly decreasing velocity. An empirical equation derived to fit the results of the Von Doenhoff calculation such that F_i is expressed as a function of U_s/U_o is

$$F_i = 0.98\left(1 - \frac{U_s}{U_o}\right)^{1/2} - 0.26 - \left[1.44\left(1 - \frac{U_s}{U_o}\right)^2 + 0.44\left(1 - \frac{U_s}{U_o}\right) - 0.007\right] \quad (1)$$

where U_s/U_o has the limits $0.898 \leqq U_s/U_o \leqq 1$. The bracketed term is neglected when the value within the brackets becomes negative $\left(0.985 \lessapprox U_s/U_o \leqq 1\right)$. Equation (1) may be converted to compressible form by the Stewartson transformations in reference 3. Since, by these transformations, the incompressible velocity is represented as the product of local Mach number and the speed of sound based upon stagnation conditions, values of U_s/U_o may be taken as values of M_s/M_o for compressible flow. (A similar use of this transformation has been made in ref. 4 in an analysis of turbulent boundary layers.) Values of F_i

are transformed to compressible form F_c by dividing by the quantity $1 + \frac{\gamma - 1}{2} M_o^2$ as indicated in reference 3. Thus, the compressible form of equation (1) is

$$F_c = \frac{1}{1 + \frac{\gamma - 1}{2} M_o^2} \left\{ 0.98\left(1 - \frac{M_s}{M_o}\right)^{1/2} - 0.26 - \left[1.44\left(1 - \frac{M_s}{M_o}\right)^2 + 0.44\left(1 - \frac{M_s}{M_o}\right) - 0.007\right]\right\} \tag{2}$$

and the limits are the same as those specified for equation (1). The relation between M_s/M_o and F_c calculated from equation (2) is shown in figure 1 for several values of M_o. The value of M_s/M_o is observed to be independent of M_o when the adverse pressure gradient begins at the leading edge $\left(F_c = 0\right)$.

In figure 2, comparisons are made of several predictions of the variation of M_s/M_o with M_o for the effective condition of the adverse pressure gradient beginning at the leading edge $\left(F_c = 0\right)$. The recent prediction in reference 5 is negligibly different from that in reference 3; consequently, one curve has been used to represent both of these predictions. Loftin and Wilson (ref. 6) used the Von Doenhoff calculations (ref. 1) and obtained results in terms of velocity decrements that are apparently identical with the analysis of this paper (see fig. 1), although their approach was somewhat different. Stewartson has made analyses (ref. 3) which indicate that his own approach may give values of M_s/M_o that are too low. Predictions that give much lower values of M_s/M_o such as that in reference 7 should, therefore, be used with caution. The prediction based upon equation (2) of this paper (and, therefore, the Loftin-Wilson prediction) would appear to be the most desirable of available methods from a practical point of view, and even this prediction is indicated later in the paper to be less conservative than might be suspected from the comparisons of figure 2. A particular shortcoming of all the analytical approaches presented and discussed herein is the neglect of the effects of the interaction of the outer flow and shock with the boundary layer. In reality, the interaction of the boundary layer and outer flow is always present and it is through this medium of interaction that separation occurs. The degree to which these analytical predictions suffer from the neglect of this interaction is, at present, unknown.

Also shown in figure 2 are the values of M_s/M_o corresponding to the maximum flow deflection through an oblique shock and the values for a normal shock. Similar curves are included in most of the subsequent figures. The implication here is that near $M_o = 1$ a normal shock will not necessarily separate a laminar boundary layer.

Pressure Rise and Flow Deflection

Two quantities that are of primary interest in the separation phenomena are the pressure rise causing separation and the deflection of the flow associated with this pressure rise. Prior to calculation of these quantities, however, certain aspects deserve consideration. Inherent in all the analytical approaches that have been mentioned is the assumption that the adverse pressure gradient begins as a discontinuity in the slope of the velocity distribution or pressure distribution. In view of this assumption, it is confusing to note the paradoxical restriction stated in some analyses that the prediction holds only for shock-free flow. More properly considered, the various analyses do not admit of the particular alterations to the boundary layer resulting from the phenomena of shock—boundary-layer interaction, but do admit of the presence of nonisentropic compressions and shocks and their accompanying rises in pressure. It follows, therefore, that the shock equations may be used to calculate the values of pressure rise and flow deflection that correspond to the values of M_s/M_o given by the analyses. From reference 8, the pressure rise that is associated with shock-induced separation of the laminar boundary layer in supersonic flow is observed to occur somewhat gradually as compared with the very steep pressure rise accompanying separation of a turbulent boundary layer. However, for the range of Mach numbers and values of M_s/M_o considered herein, the values of the static-pressure rise and flow deflection predicted by the shock equations are in excellent agreement with those calculated on the basis of exact isentropic compression at the lower Mach numbers and are in fair agreement at the higher Mach numbers. (Although this agreement may be readily confirmed by calculation, it follows logically from Busemann's airfoil theory.) Since, at the higher Mach numbers, the pressure rise accompanying separation is undoubtedly not isentropic, the shock equations may be used to predict the pressure rise and the flow deflection for the gradual as well as the abrupt pressure rise that accompanies separation.

The shock equations may be combined to give the static-pressure ratio across an oblique shock as

$$\frac{p_1}{p_o} = \frac{-M_o^2(\gamma + 1)(K^2 - 1) \pm \left\{M_o^4(\gamma + 1)^2(K^2 - 1)^2 + 4\left[K^2M_o^2(\gamma - 1) + 2\right]\left[M_o^2(\gamma - 1) + 2\right]\right\}^{1/2}}{2K^2M_o^2(\gamma - 1) + 4} \tag{3}$$

where $K \equiv M_1/M_o$. The positive sign is to be used with the bracketed term.

The associated deflection of the flow is given by

$$\phi = \tan^{-1}\left\{\frac{\epsilon - 1}{\gamma M_o^2 - \epsilon + 1}\left[\frac{2\gamma M_o^2 - (\gamma - 1) - (\gamma + 1)\epsilon}{(\gamma + 1)\epsilon + (\gamma - 1)}\right]^{1/2}\right\} \tag{4}$$

where $\epsilon \equiv p_1/p_o$. The pressure-rise coefficient P is obtained simply from

$$P = \frac{2(\epsilon - 1)}{\gamma M_o^2} \tag{5}$$

Equations (3), (4), and (5) have been applied to equation (2) $\left(F_c = 0\right)$ of this report and to the Stewartson prediction (ref. 3), the values of M_s/M_o being substituted for K. The results are presented in figure 3 and the differences are seen to be large. As has been pointed out, the results from equation (2) will be indicated to be preferable; even so, the values of ϕ and P_s from equation (2) may be too large and, subject to more experimental evidence than is now available, they should properly be regarded as only probable upper limits for the particular case of the adverse pressure gradient beginning at the leading edge and having a laminar boundary layer over the entire region of separation and interaction. Reference curves for the pressure rise through a normal shock and through the maximum oblique shock are included in figure 3, as is the curve denoting the maximum flow deflection through an oblique shock. The implication of figure 2, that near $M_o = 1$ the pressure rise through a normal shock does not necessarily separate the boundary layer, is seen directly. Alternatively, the tangency point of the flow-deflection curves with the curve for ϕ_{max} indicates that, below the value of M_o corresponding to the tangency point, the flow may be deflected at an angle greater than ϕ_{max} without separation.

Pressure Rise Causing Separation and at Separation

In addition to the overall pressure rise accompanying separation, two characteristic points of interest exist in the pressure distribution associated with shock—boundary-layer interaction that produces separation. The first of these is the pressure rise at the separation point, and the second is the higher so-called first-peak pressure rise downstream of the separation point (such as that obtained in tests with concave corners and forward-facing steps in refs. 8 and 9). Insofar as the boundary layer only is concerned, the pressure rise at the separation point is rigorously the pressure rise causing separation. However, when consideration is extended to include shock—boundary-layer interaction and the external stream, it may be somewhat misleading to say that the pressure rise at separation is that causing separation. Because of the phenomena of shock—boundary-layer interaction, the pressure rise at the separation point is followed by the higher pressure rise at the first peak, and this peak pressure rise agrees closely with the pressure rise across the compression or shock generated in the external flow as a result of deflection of the external flow by separation. In reality, therefore, unless the shock strength or peak pressure rise is sufficiently large, the upstream pressure gradient that is created through the medium of shock—boundary-layer interaction will not be large enough to cause separation. It follows that the peak pressure rise that may occur without causing separation may exceed the pressure rise at the separation point that occurs when the peak pressure rise is sufficient to cause separation. Consequently, experimental values of pressure rise obtained at the separation point cannot be considered satisfactory for determining the shock strength or pressure rise in the external flow that is associated with separation. Thus, from the broader consideration and for practical applications, it appears more appropriate to regard either the peak pressure rise or the minimum overall pressure rise with separation as the pressure rise causing separation, provided, of course, the flow is laminar over the entire region of separation.

Effect of Reynolds Number

The effect of Reynolds number upon the pressure rise causing separation of laminar boundary layers is not clearly established from a quantitative viewpoint, and the qualitative predictions differ considerably. (See refs. 8 to 11.) Sample calculations by the method of Gadd (ref. 9), which allows for effects of Reynolds number and Mach number, are given in figure 4 where $R_{x_{sp}}$ is the Reynolds number based on the distance to the beginning of separation. These curves apply only to the condition for which the boundary layer is laminar over the entire region of separation and interaction; for this condition the experimental results of references 8 and 9 tend to confirm the predicted values except near $M_o = 4$ where the prediction apparently deteriorates, as pointed out in reference 9. Although all methods proposed to date for the prediction of the pressure rise causing separation of laminar boundary layers in supersonic flow indicate an effect of Reynolds number, a similarity to the incompressible case is worth noting. In incompressible flow there is no effect of Reynolds number upon the pressure rise for laminar-boundary-layer separation so long as the surface-pressure distribution remains constant. (See refs. 12 and 13.) Inasmuch as the Stewartson transformations do not involve Reynolds number, it appears logical to expect that, for the same condition, there will be no effect of Reynolds number at supersonic speeds. However, it should be stated that this reasoning neglects the possible effects that shock—boundary-layer interaction may have upon the effects of Reynolds number.

When the boundary layer is laminar at the start of separation but not laminar over the entire region of separation and interaction, the surface-pressure distribution downstream of the laminar-separation point may change shape depending upon the location of the point at which transition to turbulent flow occurs. Reference 8 gives a clear description of these phenomena for the case of separation caused by an impinging shock. The results of reference 8 demonstrate that the maximum pressure rise associated with separation can be much greater when transition occurs over the separated region than when the flow remains laminar over the entire region.

Figure 5 presents experimental results obtained in the Langley 9-inch supersonic tunnel at $M_o = 2.41$ where separation was produced by a two-dimensional forward-facing step mounted on a flat plate. The step was 0.060 inch high and located 4 inches from the leading edge of the plate. The step height exceeded the theoretical laminar-boundary-layer thickness beyond Reynolds numbers of approximately 0.3×10^6, based on the distance from the leading edge to the step. Although schleiren observations were not made, the presence of the characteristic laminar foot in the surface-pressure distributions (see ref. 8 and sketch in fig. 5) gave assurance that the boundary layer was always laminar at the separation point throughout the range of these tests. The data presented correspond to the peak in the laminar foot and to the first peak downstream of the laminar foot as shown in the sketch in figure 5. If the data for the peak in the laminar foot may be regarded as indicative of the pressure rise at the separation point (see refs. 8 and 9), then the order of magnitude of the pressure rise at the separation point is in general agreement with the predictions of reference 9 shown in figure 4. No direct comparison can be made because the Reynolds numbers are based on different lengths. From figure 5 one might conclude that at this Mach number the pressure rise at the separation point (properly at the laminar-foot peak) is not greatly affected by Reynolds number except for conditions where the step height is less than the boundary-layer thickness. However, the indicated overall effect of Reynolds number would occur if the variation of P with R_x were proportional to R_x raised to some negative power of the order of -0.25. The same conclusion as to the effects of Reynolds number would appear to hold for the first-peak pressure rise so long as the flow is entirely laminar. In light of the results of reference 8, transition begins to take place over the separated region near $R_{x_{step}} \approx 1.1 \times 10^6$ and, when this occurs, a rapid increase in the value of the first-peak pressure rise occurs with further increase in Reynolds number. It is important to note that this first-peak pressure rise reaches a maximum value that is in excess of that obtained with a fully turbulent boundary layer as indicated in figure 5. (A similar result has been obtained in ref. 14 with a forward-facing wedge.) At the maximum Reynolds numbers of these tests, the first-peak pressure rise appears to be falling off toward the fully turbulent first-peak value, and the laminar-foot pressure rise appears to be rising toward the fully turbulent separation-point value.

The difference in the variation with Reynolds number of the pressure rise for the laminar-foot peak and the pressure rise for the first peak downstream of the laminar foot may explain what has, in the absence of further experimental evidence, appeared to be contradictory experimental results for forward-facing steps as noted in reference 11. Although the pressure rise for the laminar-foot peak may prove to be relatively independent of the means by which separation is obtained, as tends to be indicated from comparison of the present results with those of reference 8, the pressure rise for the first peak may not have such independence. Clarification of these points must await further experimental work. At this time, the results of figure 5 should be regarded for the most part as what may take place qualitatively; quantitatively, the results may vary with the turbulence level of the test facility, with the ratio of step height to boundary-layer thickness, and with other factors.

TURBULENT BOUNDARY LAYERS

Effect of Reynolds Number

In reference 15 experimental results were presented for $M_o = 1.55$ that showed a negligible effect of Reynolds number upon the pressure rise corresponding to the first peak in the pressure distribution associated with the separation of a turbulent boundary layer by a forward-facing step. Subsequently, similar results were obtained in the investigation of reference 11 for $M_o = 3.03$. More recently, additional experimental data have been obtained that support the negligible effect of Reynolds number. A portion of these data has been published in reference 16. In figure 6 the data of reference 15 $\left(M_o = 1.55\right)$, reference 16, and some previously unpublished data are presented to show the negligible effect of Reynolds number. All these data were obtained in the same test facility (a blowdown tunnel of the Langley 9-inch Supersonic Tunnel Section), and, throughout the range of the tests, the step height was at least twice the boundary-layer thickness. (A preliminary investigation indicated no significant effect of step height when the step height was approximately twice the boundary-layer thickness or higher, and in this respect the results agreed with an examination of data discussed in ref. 15 and with the results of ref. 17.) In view of these results, Reynolds number effects are neglected in the following sections.

Pressure Rise Causing Separation and at Separation

Before proceeding further, some distinction between the pressure rise at separation and the pressure rise causing separation should be made for turbulent boundary layers as was made for laminar boundary layers. The characteristic pressure distributions associated with separation of turbulent boundary layers do not have the inherent clear differentiation between the pressure rise at the separation point and the peak pressure rise as do those for laminar boundary layers whose laminar foot is easily distinguished from the following first peak. Nevertheless, sufficient experimental studies have been made with turbulent boundary layers to show that separation occurs before the peak pressure rise is reached - for example, the results of tests at $M_o = 2.92$ given in reference 17. From these results the following observations may be made. First, for the data from forward-facing steps for which the effects of step height are essentially eliminated (see figs. 4 and 5 of ref. 17), the value of P at the separation point is about 0.17 as compared with about 0.26 at the peak. Second, the peak pressure rise for separation produced by a forward-facing step (about 0.26) is observed to be of the same order of magnitude as the maximum overall pressure rise that can be imposed by an impinging shock without creating separation (about 0.29). Third, when an impinging shock is strong enough to cause separation, the pressure rise at the separation point is also about 0.17 as was obtained for the forward-facing step. (The initial "knee" in the characteristic pressure distribution for impinging shocks which is indicative of separation is not defined sharply; nevertheless, there is no question that the pressure rise at the separation point is well below the maximum pressure rise that can be obtained without separation. See figs. 12 and 14 of ref. 17.) Fourth, the peak pressure rise for forward-facing steps, the pressure rise at the concave corner beyond the inflection point in the characteristic pressure distribution for separation caused by impinging shocks, the maximum pressure rise that can be imposed by an impinging shock without causing separation, and the overall pressure rise through a theoretical inviscid simple reflection of the maximum strength impinging shock that will not cause separation are in fair agreement, all being within the limits from about 0.26 to 0.29. Thus, by accounting for the condition of simple reflection, the peak pressure rise obtained from forward-facing steps may be used to calculate the approximate minimum strength of the impinging shock that will cause separation. The exact minimum strength of the impinging shock that will cause separation may be slightly higher than this approximate value in view of the differences previously indicated between the peak pressure rise and the overall pressure rise necessary to cause separation (about 0.26 as compared with about 0.29).

From the previous observations it becomes apparent that the pressure rise at the separation point cannot be used to gage the strength of the shock in the external flow that is necessary to cause separation. Thus, from the overall consideration of the external flow, shock—boundary-layer interaction, and the boundary layer, it appears more appropriate for practical applications to regard either the peak pressure rise or the minimum overall pressure rise with separation as that causing separation.

Analytical Considerations

A number of analytical studies have been made of the pressure rise causing turbulent-boundary-layer separation in supersonic flow. (See refs. 4, 18, 19, and 20, for example.) Inasmuch as the methods of Reshotko and Tucker (ref. 20) and of Mager (ref. 4) lend themselves to rapid application and do, in fact, admit of a possible range of pressure rises within which the values given by more elaborate procedures would fall, these two methods have been selected for consideration here. Both methods hinge upon the incompressible values of the boundary-layer form parameter H_i chosen for the initial flow and for the flow at separation. Once the values of H_i are selected, the corresponding values of M_s/M_o $\left(\text{or } M_1/M_o\right)$ are readily obtainable. The values of P and ϕ may be calculated by equations (3), (4), and (5) of this paper. (Ref. 20 has pointed out that use of the linearized shock equations in ref. 4 is unsatisfactory.)

The initial value of the form parameter H_{i_o} generally ranges from 1.222 (a 1/9-power profile) to 1.400 (a 1/5-power profile). The separation value of the form parameter H_{i_s} is indicated from references 21 to 23 to range between 1.8 and 2.8. From these values of H_{i_o}

and H_{i_s}, the maximum and minimum combinations are $H_{i_o} = 1.222$ and $H_{i_s} = 2.8$, and $H_{i_o} = 1.400$ and $H_{i_s} = 1.8$, respectively. The values of the pressure-rise coefficient corresponding to these combinations according to the methods of Mager and of Reshotko and Tucker are given in figure 7. Also shown are the prediction of Gadd (ref. 9), the laminar-boundary-layer curves from equation (2) for $F_c = 0$ and from the Stewartson prediction, the experimental compilation of Schuh for the pressure rise at the separation point (ref. 24), the prediction of Schuh as given by the Stewartson transformation for $H_{i_o} = 1.222$ (Schuh's approach admits of only ∞ for H_{i_s}), and, for comparison with Schuh's prediction and to show the effect of increasing H_{i_s} from 2.8 to ∞, the predictions by the methods of Mager (ref. 4) and of Reshotko and Tucker (ref. 20) for $H_{i_o} = 1.222$ and $H_{i_s} = \infty$. In addition, the reference curves for the normal shock and the maximum oblique shock are given, the implication again being that a normal shock may not separate the boundary layers at low Mach numbers. The investigations of references 25 and 26 have shown that, at Mach numbers near 1, a normal shock does not necessarily cause separation.

If the predictions of figure 7 are to be judged solely on their ability to predict the pressure rise at the separation point, then the transformed prediction of Schuh and the closer prediction of Gadd are to be rejected; the predictions of Mager and of Reshotko and Tucker are suitable for combinations of H_{i_o} and H_{i_s} which yield values of M_s/M_o of the order of 0.85. (For $H_{i_o} = 1.400$ and $H_{i_s} = 1.8$, the method of Mager gives $M_s/M_o = 0.862$, whereas that of Reshotko and Tucker gives $M_s/M_o = 0.874$.) If these methods are advanced so as to admit of all pressure rises associated with separation (at separation point, peak, maximum, etc.), then the highest and lowest curves for turbulent boundary layers given in figure 7 might be crudely indicative of the range in which such pressure rises may lie. It is apparent from figure 7 that changing the value of H_{i_s} from 2.8 to ∞ has no large effect on the predicted results. The fact that Schuh's experimental compilation for the turbulent separation point is generally lower than Stewartson's prediction for laminar boundary layer does not imply beyond doubt that this laminar prediction is too high, because the laminar prediction is an upper limit prediction $\left(F_c = 0\right)$ for the pressure rise causing separation, and Schuh's compilation is at best a probable lower limit for turbulent separation; however, one suspects that Stewartson's prediction may be high from this comparison and from the reasons given in the first part of this paper. Accordingly, the prediction of equation (2) for $F_c = 0$

probably gives a more reliable upper limit for wholly laminar flow. In comparison with Schuh's compilation, the predictions of Gadd in figure 4 appear to have the proper order of magnitude since turbulent boundary layers are known to be capable of withstanding roughly three times the pressure gradient that a wholly laminar boundary layer can resist. (See ref. 27, for example.)

The two-dimensional-flow deflections through an oblique shock have been calculated for most of the curves of figure 7 and are shown in figure 8. As previously stated, the tangency point of the individual curves with the curve for ϕ_{max} indicates that, below the value of M_o corresponding to the tangency point, the flow may be deflected without separation at an angle greater than ϕ_{max}.

Peak Pressure Rise

The pressure rise corresponding to the first peak in the surface-pressure distribution associated with separation, generally referred to as peak pressure rise (see refs. 11 and 19, for example), is of particular interest in that it has been found to give fair predictions of loading associated with separation and of deflection of the flow outside the separated region. Figure 9 presents the experimental variation of the peak pressure-rise coefficient with Mach number as obtained from figure 6 and compares this variation with that which was given in reference 15 and based on an experimental compilation. Also shown is the prediction by the method of Reshotko and Tucker, whose assumption $H_{i_o} = 1.286$ and $H_{i_s} = 2.2$ leads to $M_1/M_o = 0.762$. The curve given by the empirical relation

$$\bar{P}_f = \frac{3.2}{8 + \left(M_o - 1\right)^2} \tag{6}$$

is also included; this relation implies that M_1/M_o is not constant. The curve given in reference 15 is seen to be in excellent agreement with the present experimental results except at the higher Mach numbers where the disagreement may be attributed for the most part to the use of an experimental point at $M_o = 3.03$ from reference 10 which has been shown by Lange (ref. 11) to be inaccurate. Equation (6) depicts the experimental variation closely. The Reshotko-Tucker prediction is slightly high at the higher Mach numbers; however, up to Mach numbers of 4, this amount of overprediction is not important for engineering purposes. Figure 10 presents the two-dimensional-flow deflections through an oblique shock that correspond to the curves in figure 9. The differences at the higher Mach numbers are again evident. In view of the comparisons of figures 9 and 10, it would appear desirable to use equation (6) or the Reshotko-Tucker prediction at Mach numbers much beyond about 2.4 for the basic curve of analyses such as that given in reference 15. (The difference between the peak pressure rise and the slightly higher overall pressure rise which would cause separation may explain in part why the empirical increment of 0.06 added to the values of P_f improved the analyses of ref. 15.) In figure 11 the results of figure 9, excluding the curve from reference 15, are presented in the form of static-pressure ratio inasmuch as this form is sometimes found more convenient for practical applications. In addition to the reference curves corresponding to the maximum deflection through an oblique shock and to the normal shock limit, figures 9 to 11 also show the curve corresponding to the maximum simple reflection for an impinging oblique shock.

Effect of Ratio of Specific Heats

The effect of the ratio of specific heats γ upon the pressure rise associated with separation has received almost no direct study. Calculations of the effect for the peak pressure rise by use of the Reshotko-Tucker value of 0.762 for M_1/M_0 are given in figures 12 to 14. A peak value for helium ($\gamma = 1.667$) is available for $M_0 = 3.48$ from some small two-dimensional-nozzle investigations in the Langley 9-inch Supersonic Tunnel Section. Reynolds number effects were negligible. This value is shown in figure 12 and agrees closely with the predicted value. In reference 28, tests were conducted with rocket motors having conically divergent nozzles to study separation from the nozzle wall. The value of γ for these tests ranged from about 1.20 to 1.26. The results state that all the calculated flow deflections (corresponding to the peak pressure rise) are within $18^\circ \pm 3^\circ$. The local Mach numbers immediately ahead of separation ranged from about 2.5 to 3.3. These results afford only a rough comparison with the predictions in figure 14, but there is general agreement as to order of magnitude. (It should be noted that the Reshotko-Tucker prediction is applicable to axially symmetric flow.) If the indications of these meager comparisons can be considered typical, then the Reshotko-Tucker prediction may be considered satisfactory for estimating the effects of γ. In terms of the static-pressure ratio (fig. 13), the effect of γ is seen to disappear near $M_0 \approx 1.65$ where the effect of γ reverses.

CONCLUDING REMARKS

A study has been made of some recent contributions to the problem of shock-induced boundary-layer separation, and additional analytical and experimental results are presented. The probable ranges within which the pressure rises and flow deflections associated with separation may be expected to lie are shown. Consideration is given to the effects of Mach number, adverse pressure gradient, and Reynolds number for laminar boundary layers and to the effects of Mach number, Reynolds number, and ratio of specific heats for turbulent boundary layers.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 7, 1955.

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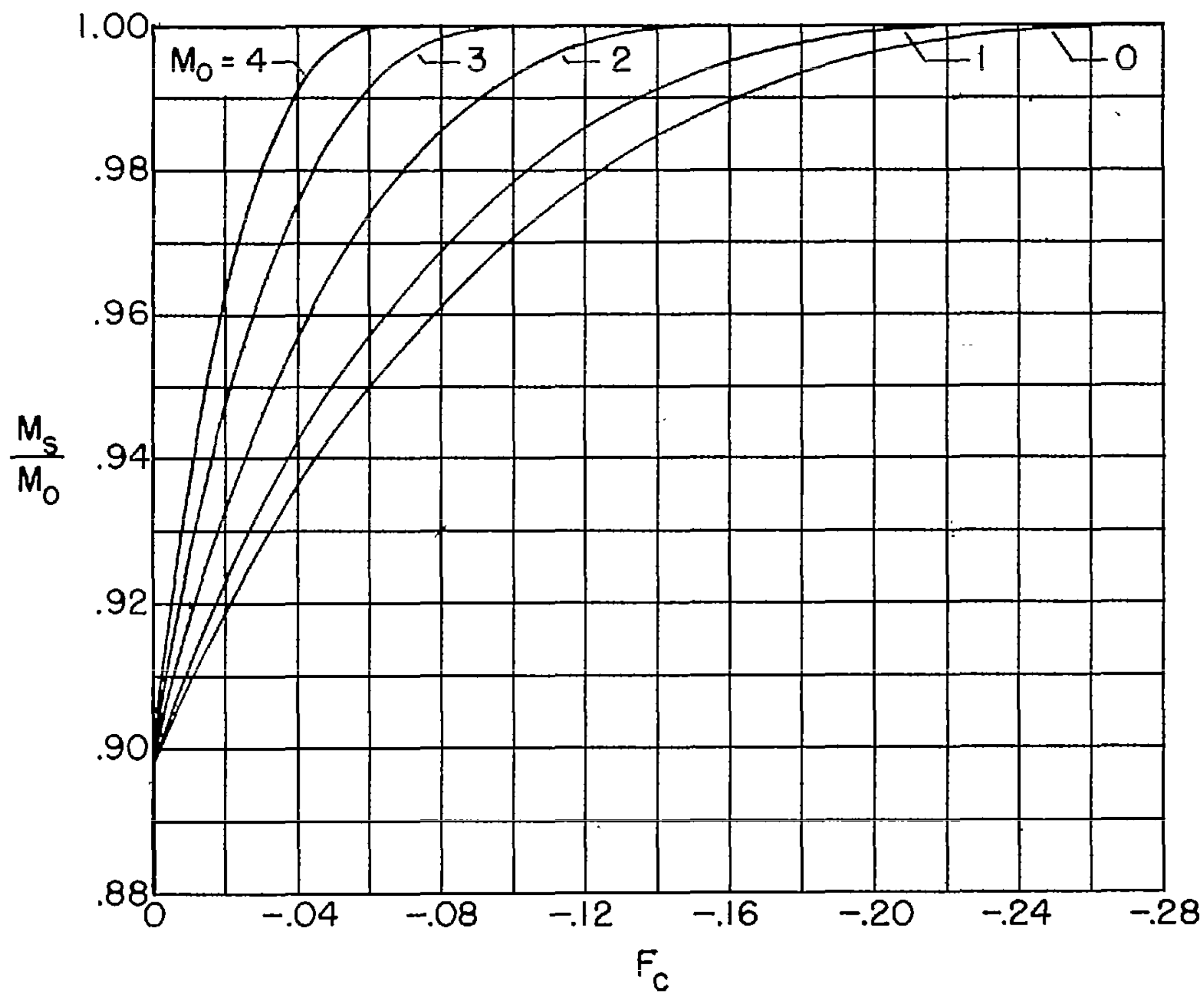

Figure 1.- Effect of free-stream Mach number and compressible nondimensional velocity gradient upon the ratio of local to free-stream Mach number causing laminar-boundary-layer separation. (From eq. (2) of present paper.)

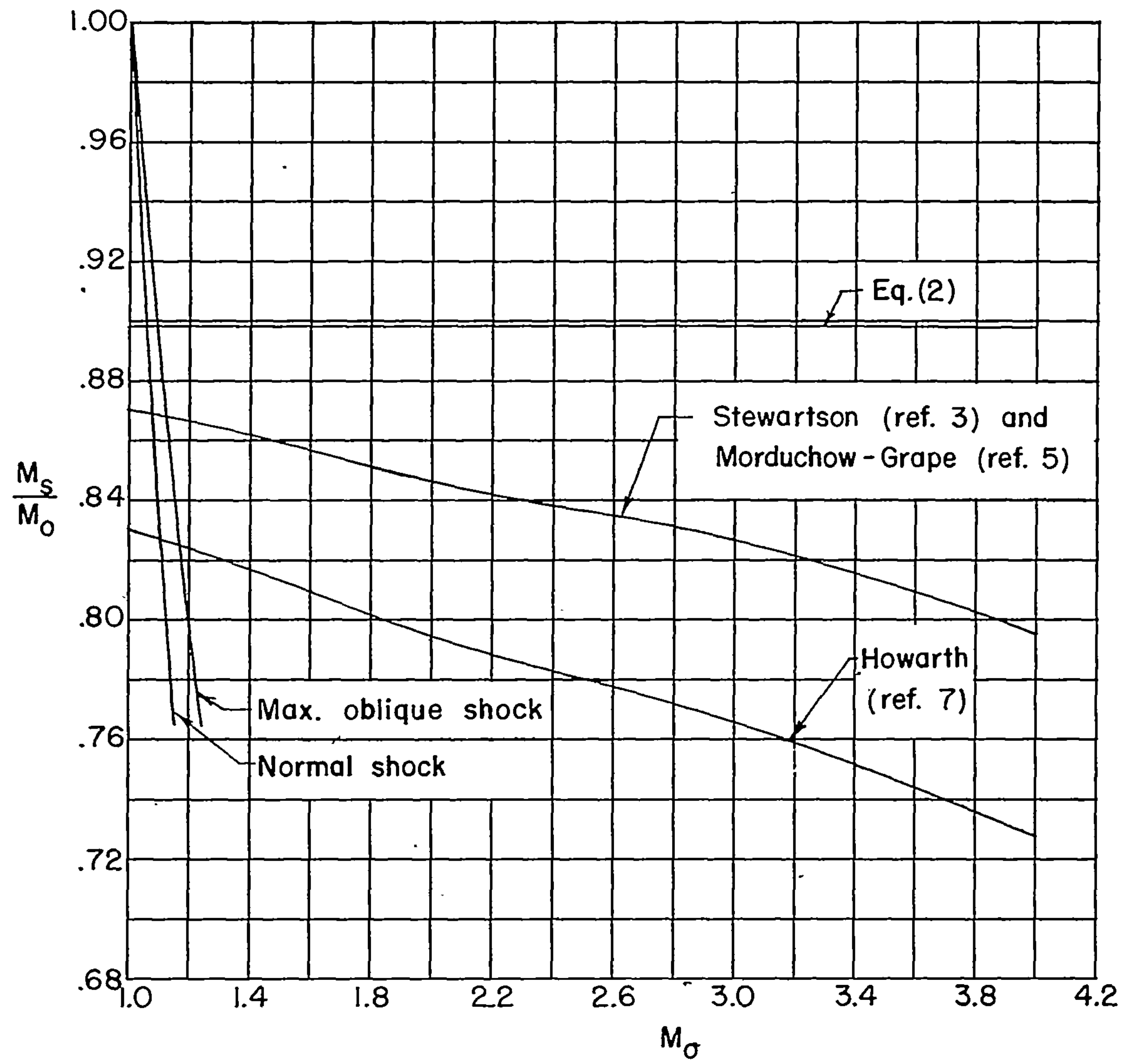

Figure 2.- Comparison of several predictions of effects of free-stream Mach number upon the ratio of local to free-stream Mach number causing laminar-boundary-layer separation. $F_c = 0$.

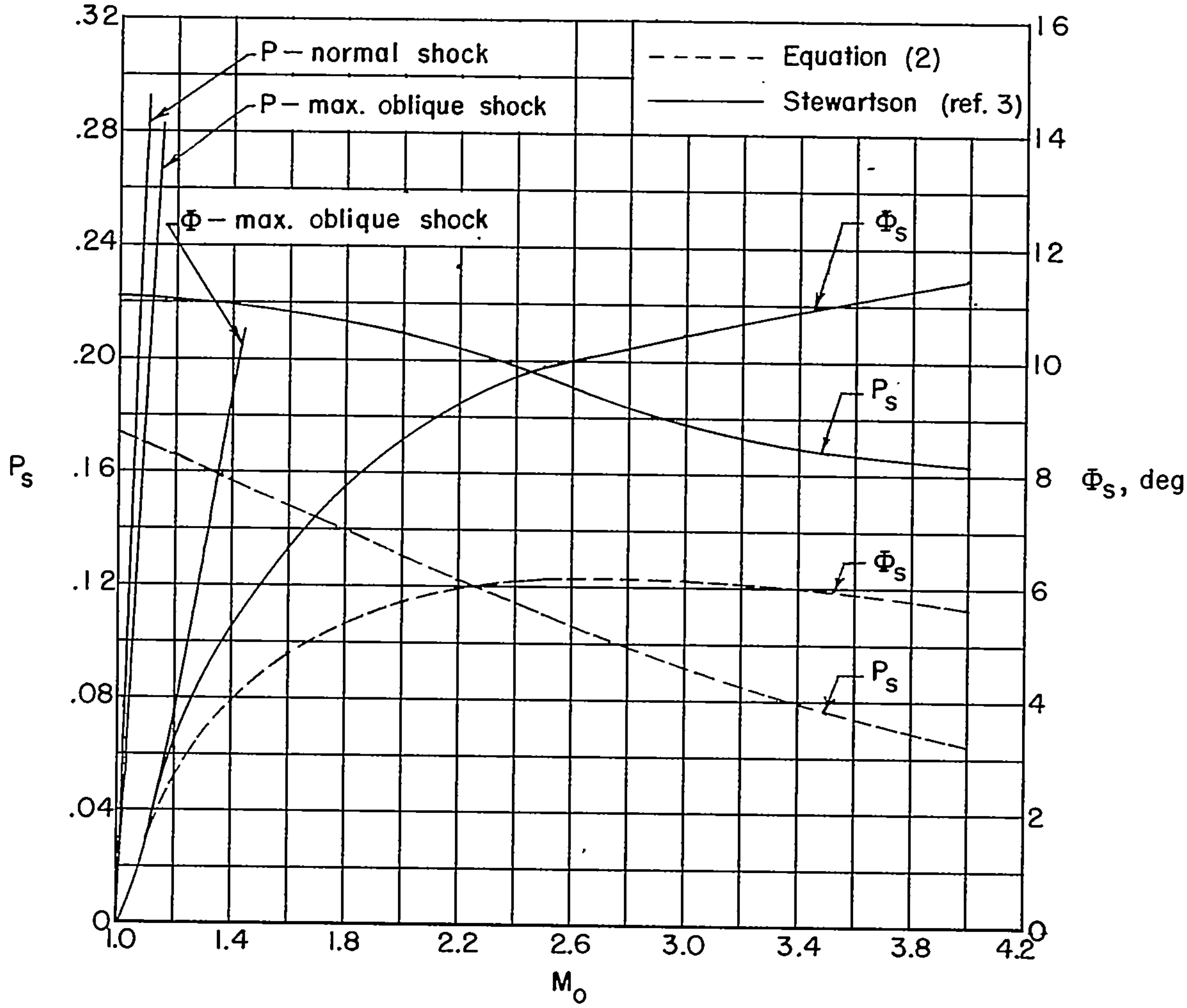

Figure 3.- Effect of free-stream Mach number upon the laminar pressure-rise coefficient causing separation and upon the two-dimensional deflection of the flow corresponding to this pressure rise. $F_c = 0$.

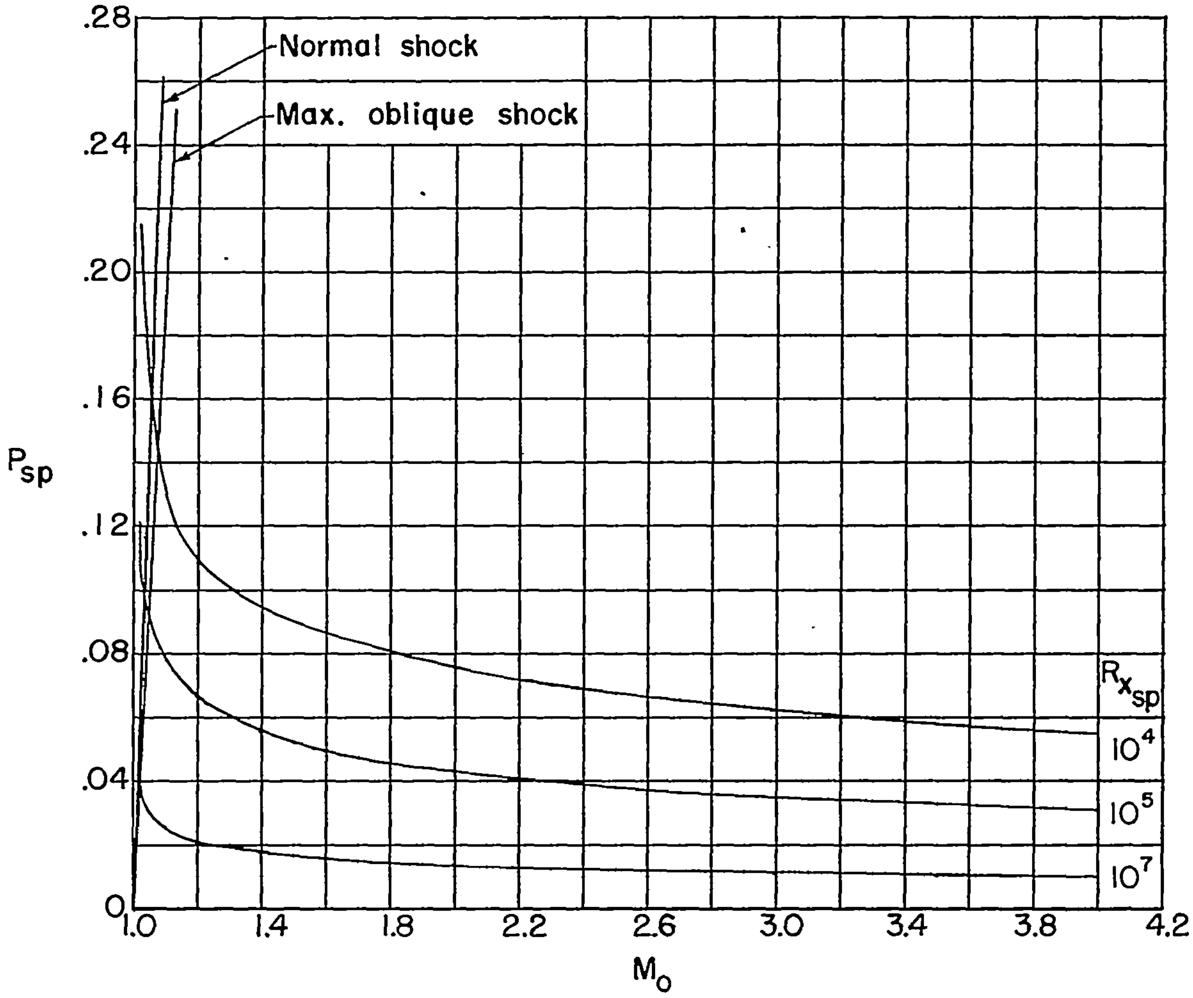

Figure 4.- Sample calculations by the method of Gadd showing the effect of Reynolds number and free-stream Mach number upon the pressure-rise coefficient at the separation point for wholly laminar boundary layers.

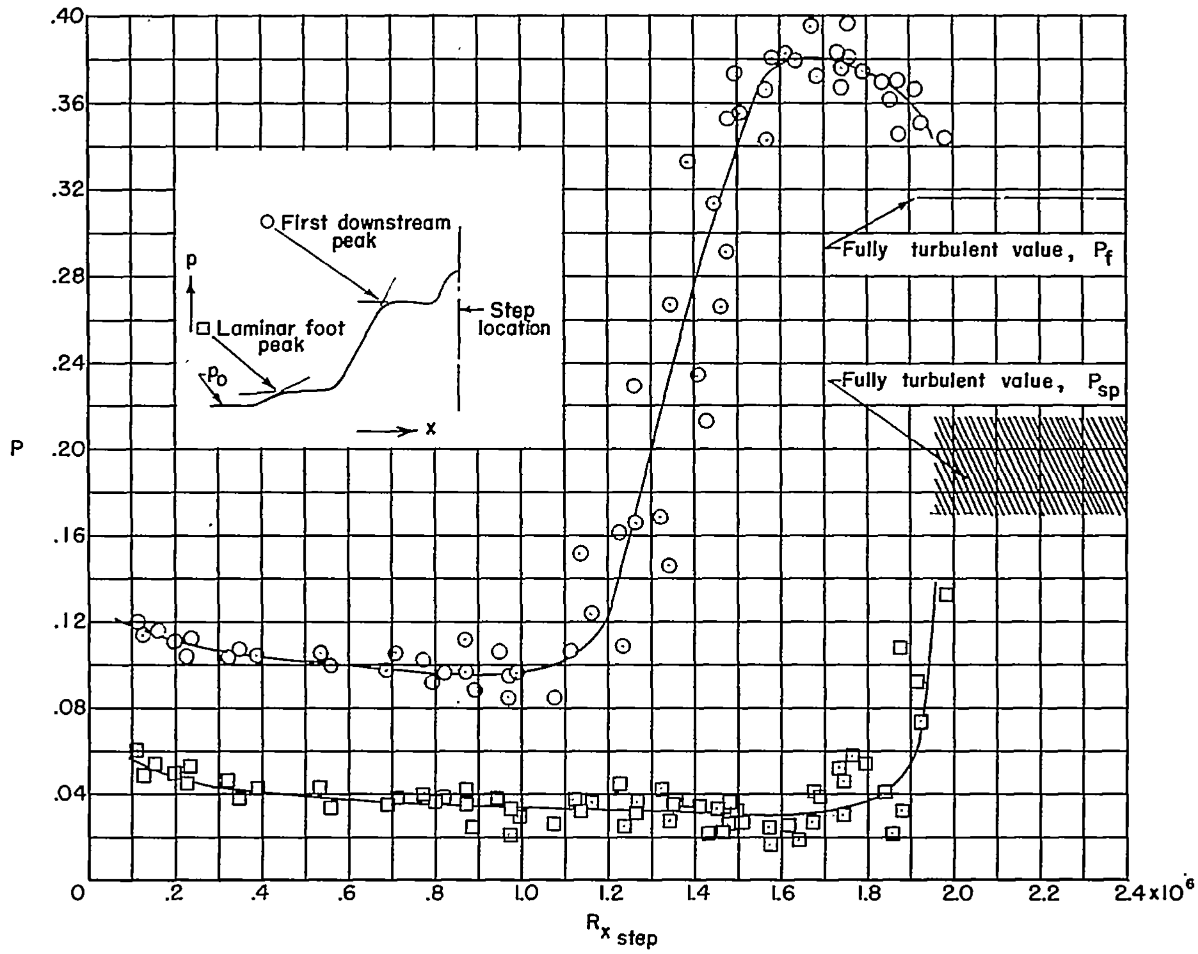

Figure 5.- Effect of Reynolds number upon the pressure-rise coefficient corresponding to the laminar-foot peak and to the first downstream peak for separation of a laminar boundary layer by a forward-facing step at a free-stream Mach number of 2.41.

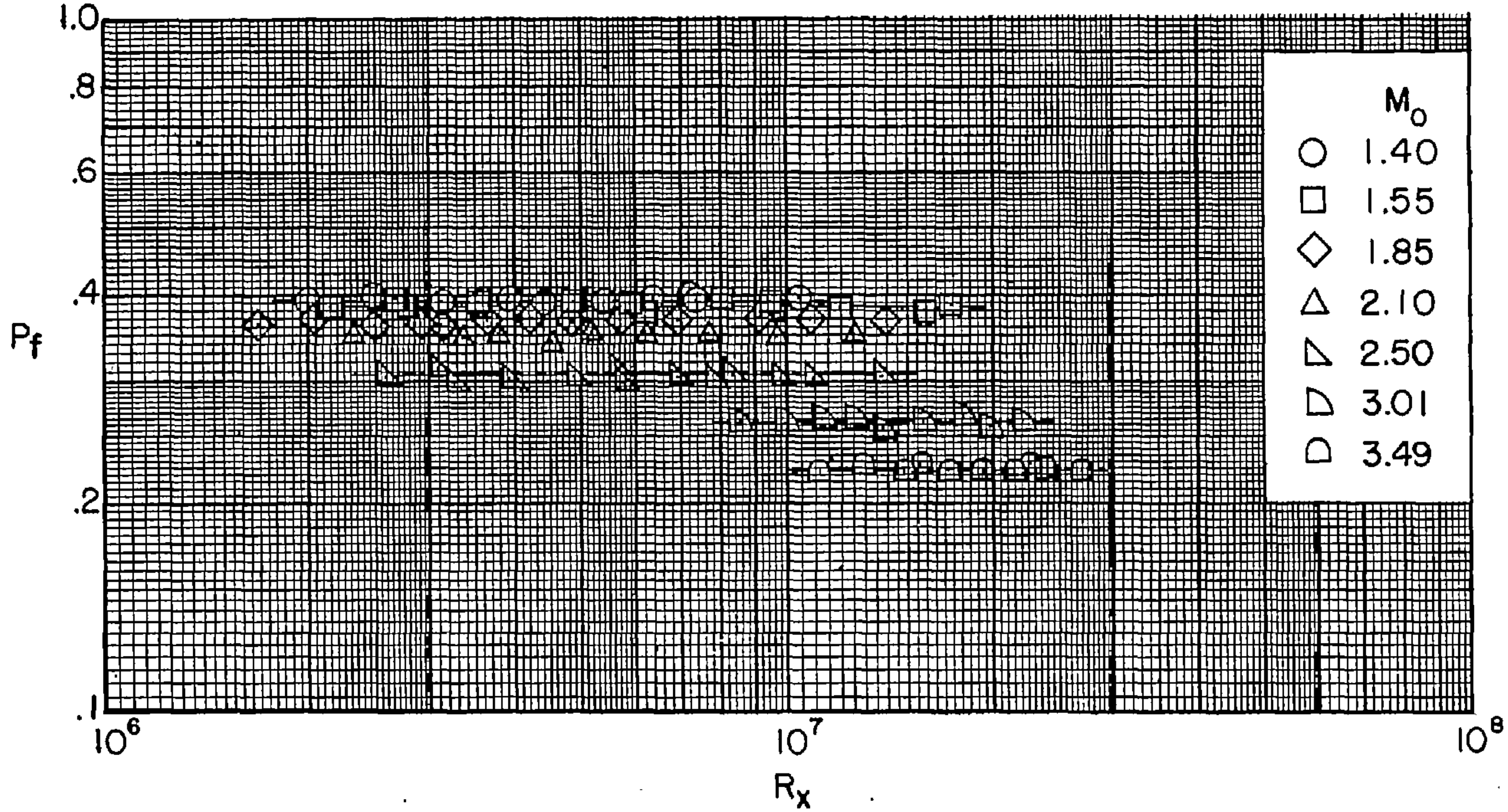

Figure 6.- Effect of Reynolds number upon the peak pressure-rise coefficient associated with the separation of a turbulent boundary layer by a forward-facing step.

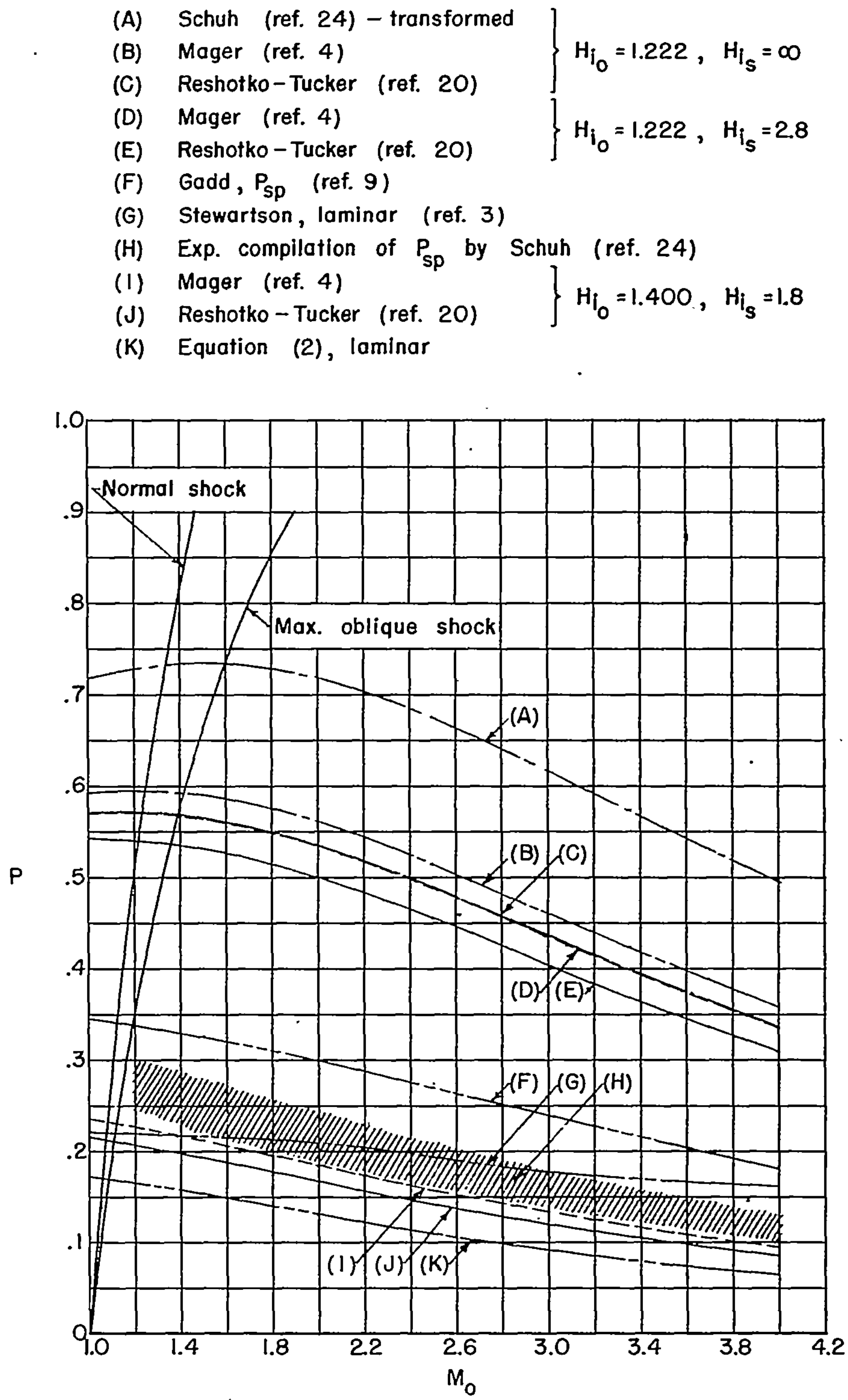

Figure 7.- Some predictions and comparisons of pressure rises associated with boundary-layer separation. Turbulent boundary layer unless otherwise specified.

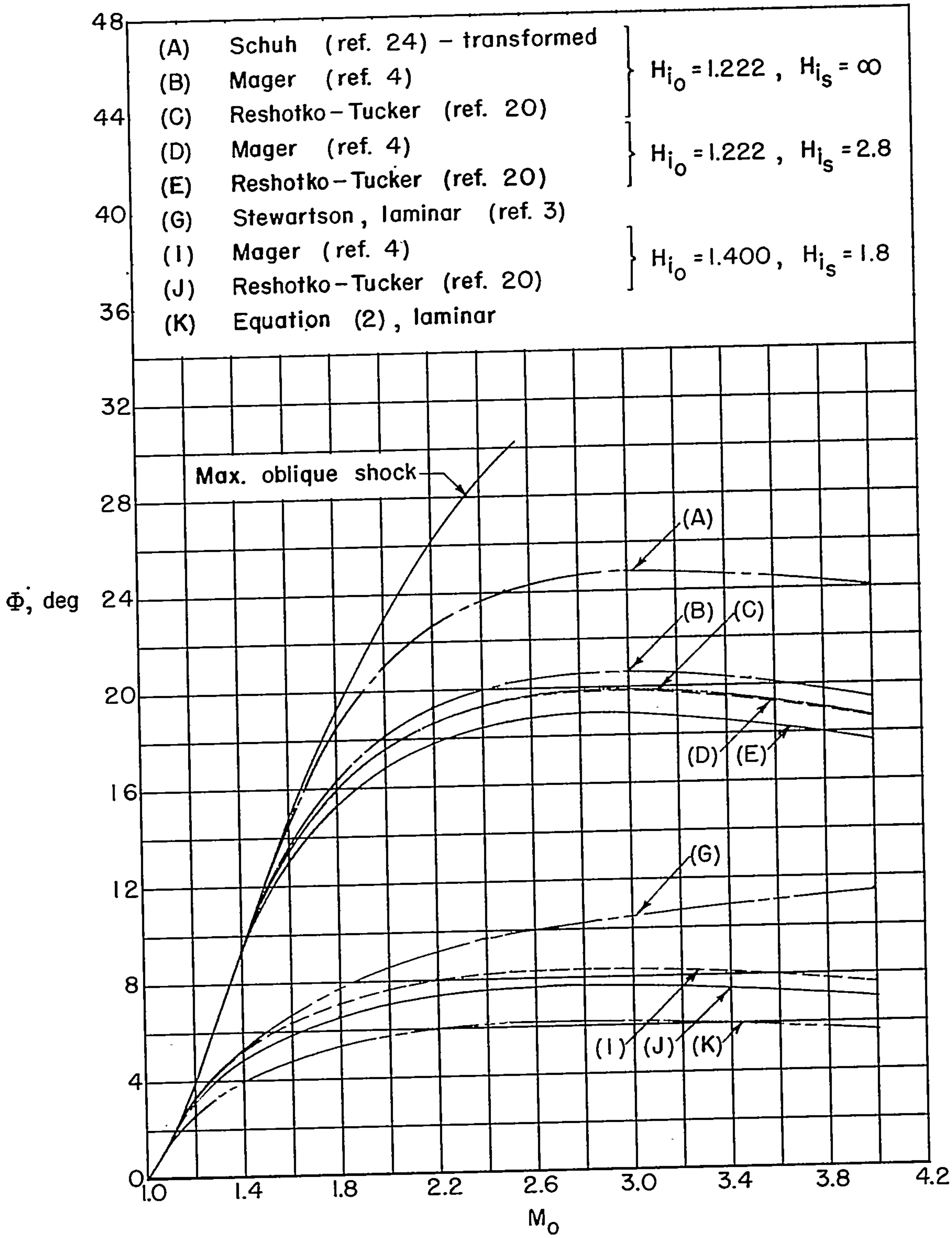

Figure 8.- Two-dimensional-flow deflections corresponding to some of the predictions of figure 7.

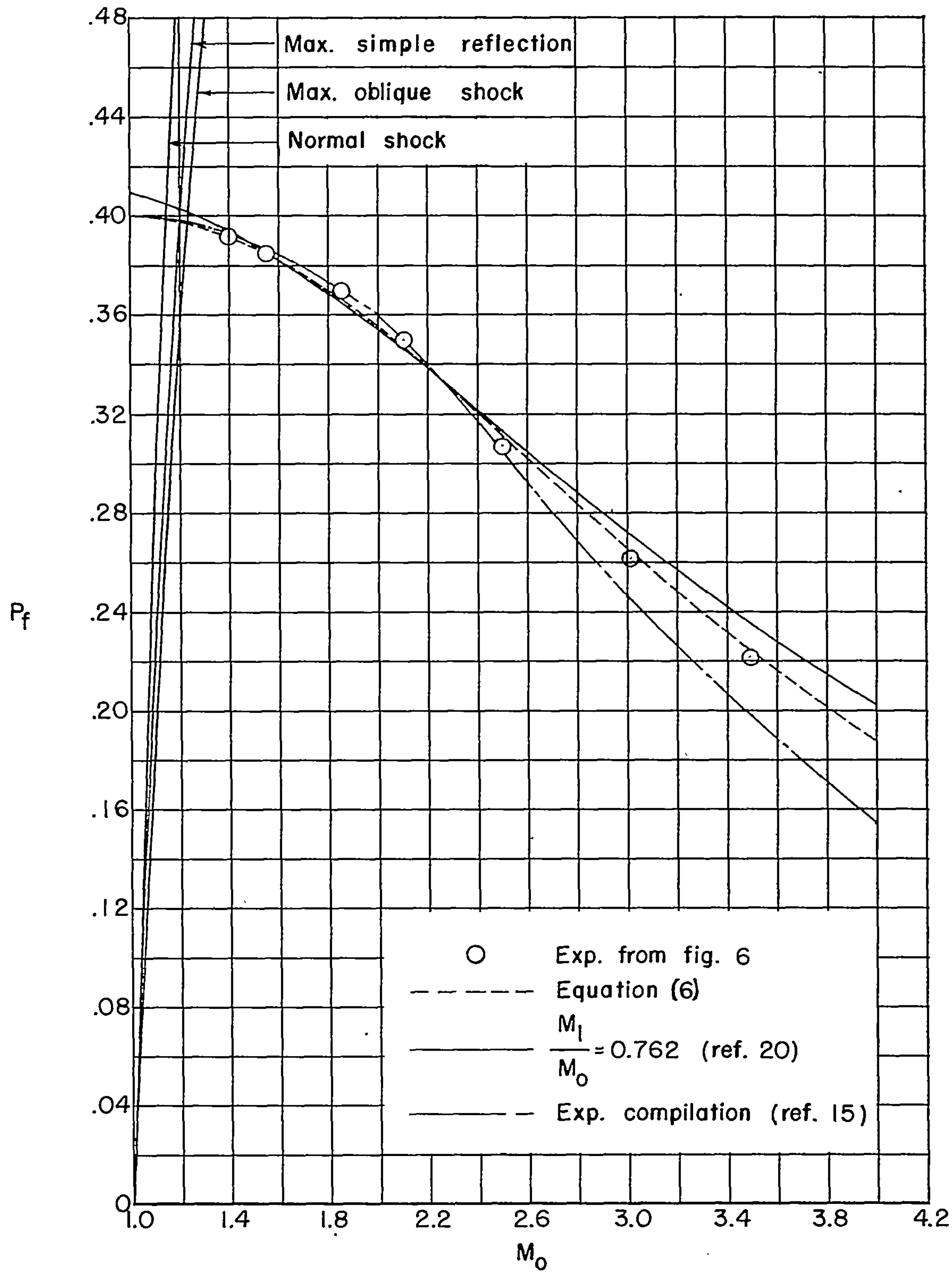

Figure 9.- Effect of free-stream Mach number upon peak pressure-rise coefficient for turbulent boundary layers.

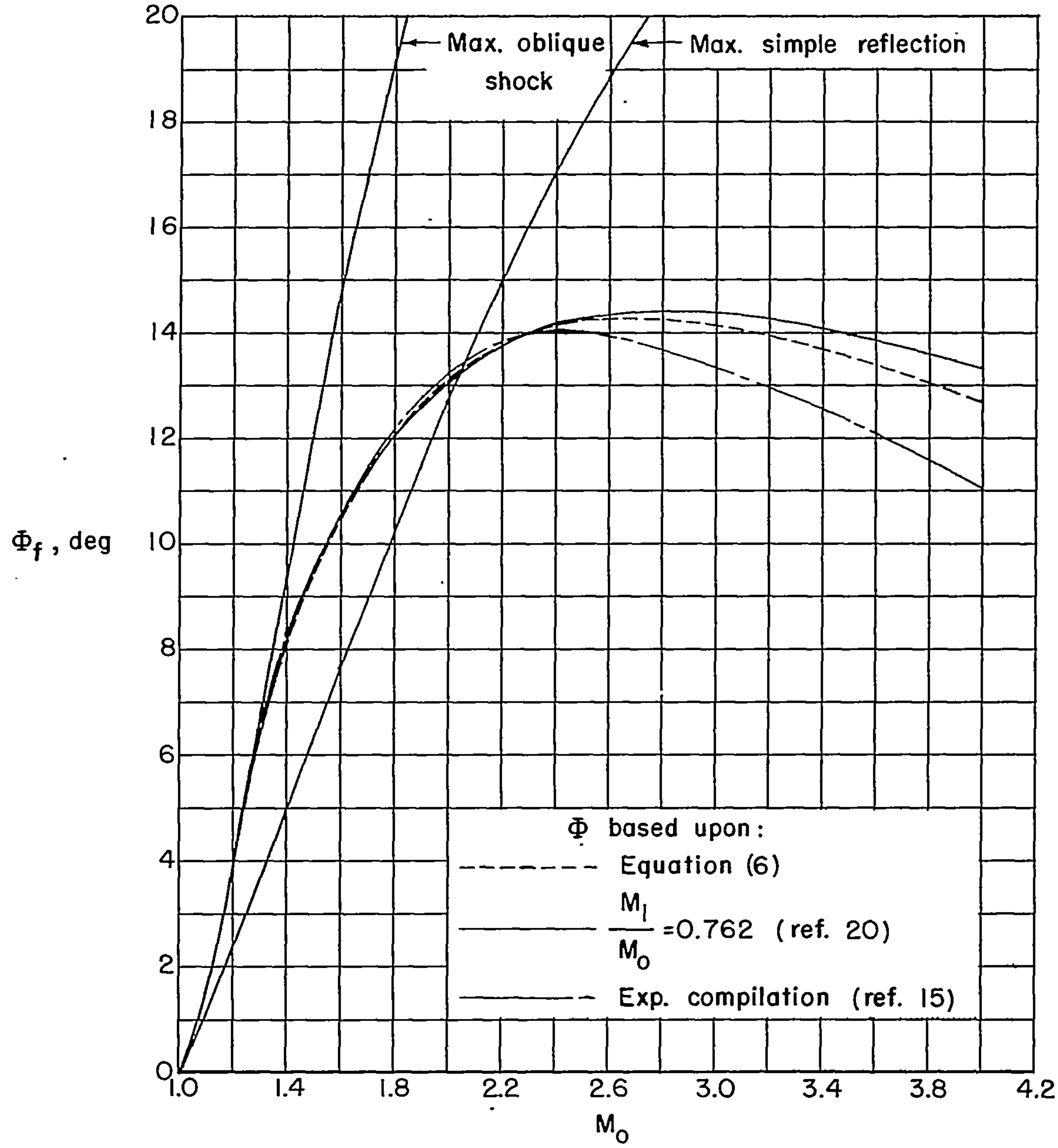

Figure 10.- Two-dimensional-flow deflections corresponding to peak pressure-rise coefficients of figure 9.

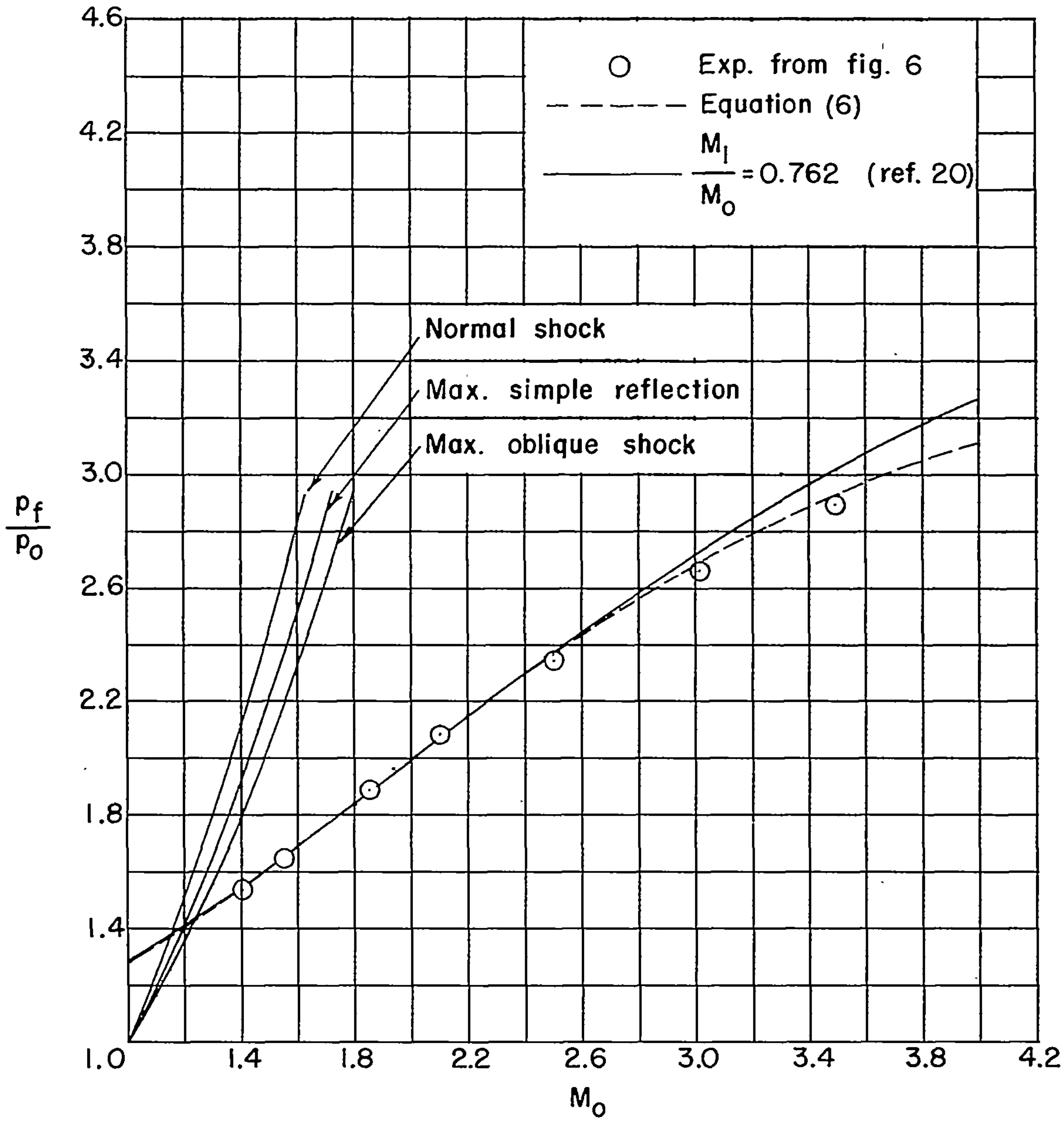

Figure 11.- Static-pressure ratios corresponding to peak pressure-rise coefficients of figure 9.

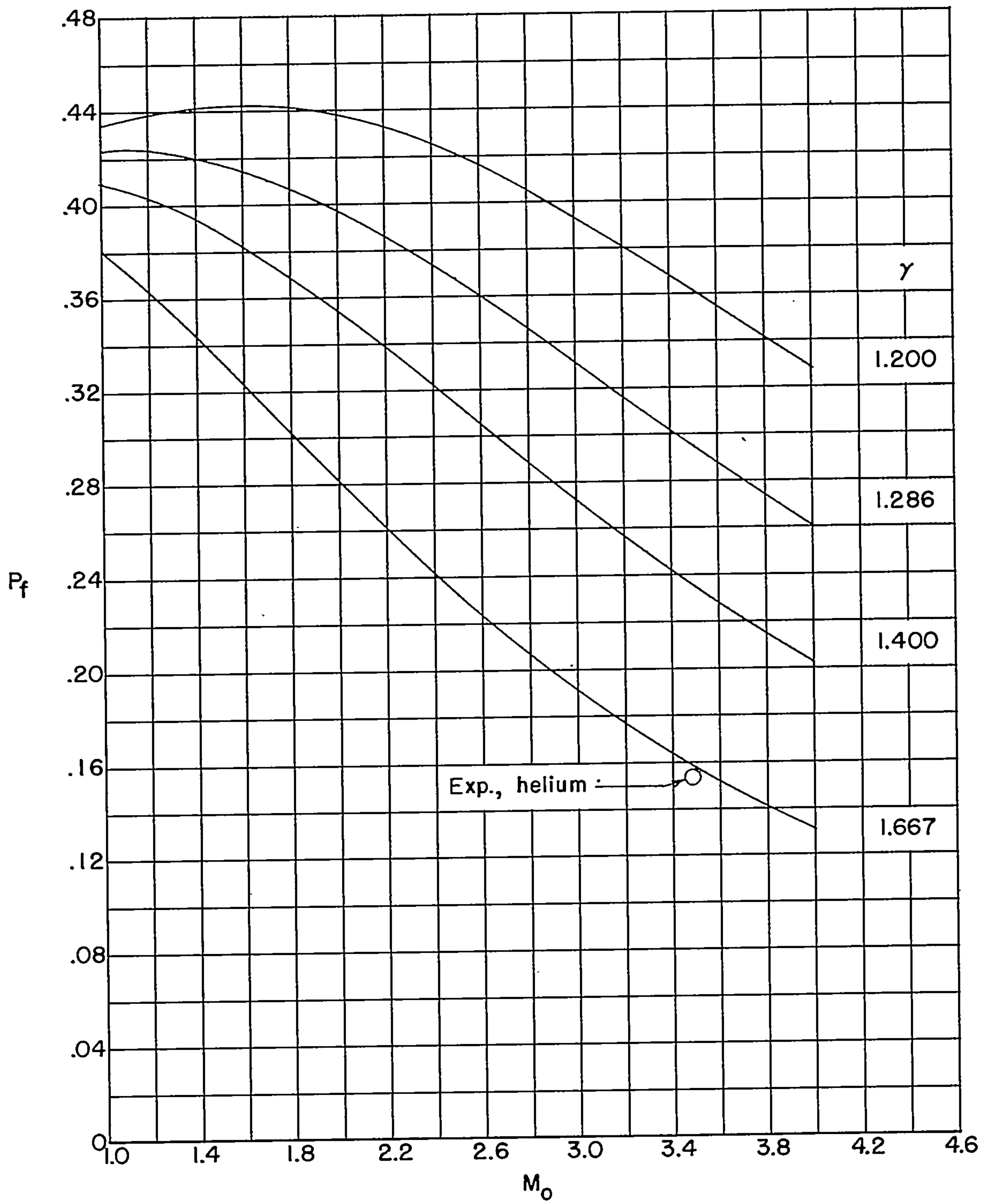

Figure 12.- Effect of ratio of specific heats upon peak pressure-rise coefficient for turbulent boundary layers. Reshotko-Tucker prediction, $\frac{M_1}{M_o} = 0.762$.

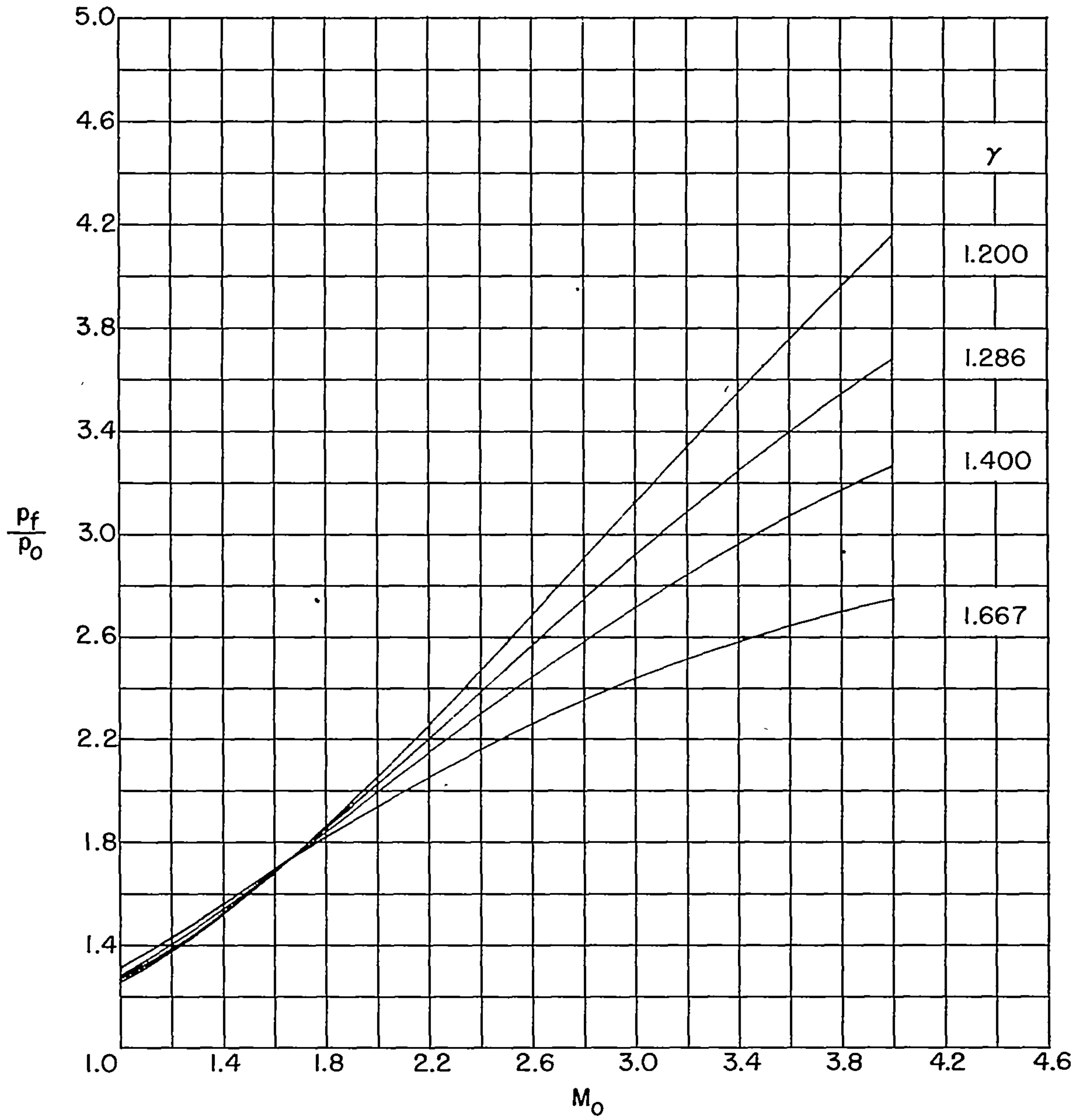

Figure 13.- Static-pressure ratios corresponding to peak pressure-rise coefficients of figure 12.

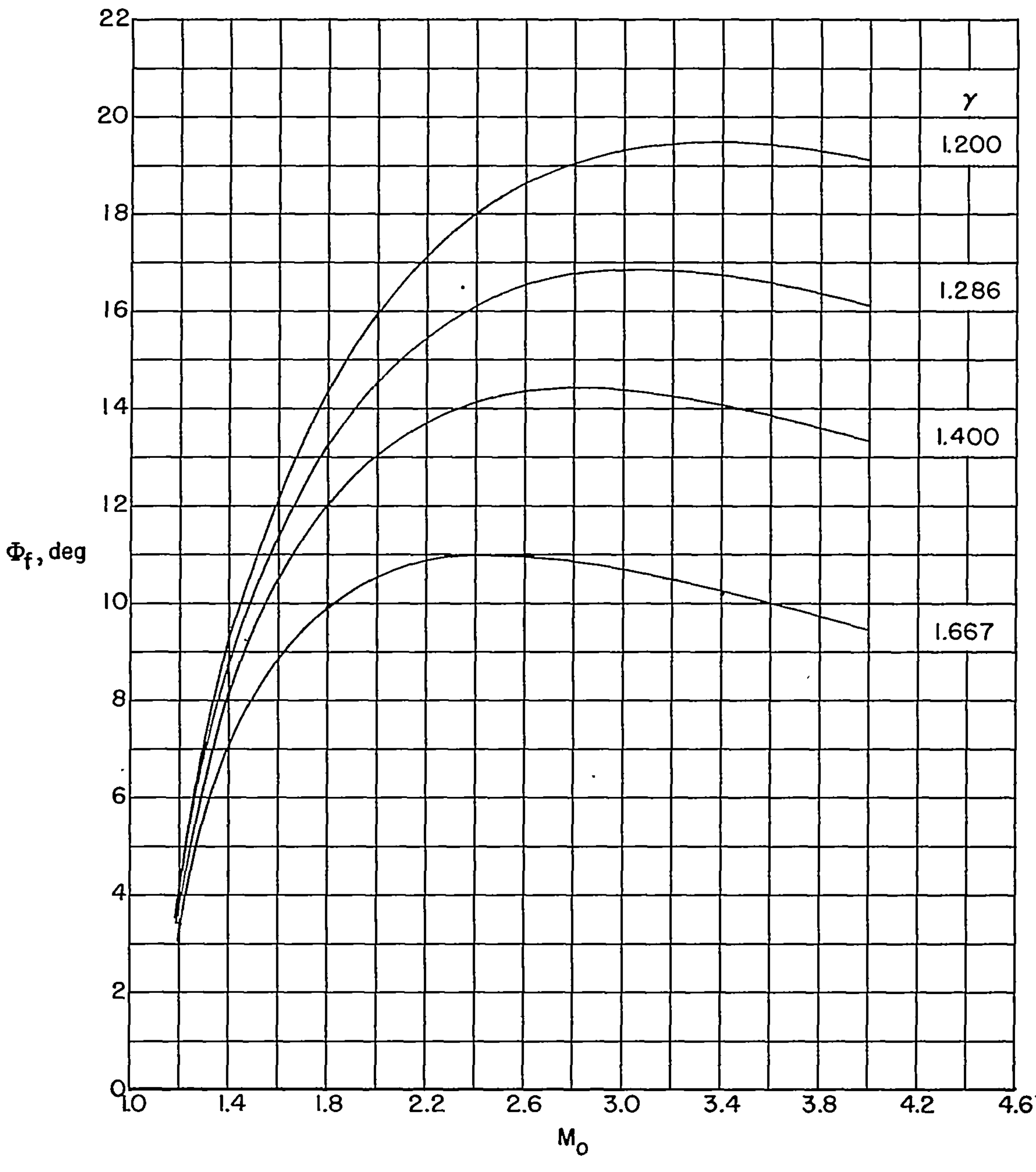

Figure 14.- Two-dimensional-flow deflections corresponding to peak pressure-rise coefficients of figure 12.